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Matrix Realization of Gauge Theory on Discrete Group Z_2

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Abstract

We construct a 2×2 matrix algebra as representation of functions on discrete group Z_2 and develop the gauge theory on discrete group proposed by Starz in the matrix algebra. Accordingly, we show that the non-commutative geometry model built by R.Conquereax, G.Esposito-Farese and G.Vaillant results from this approach directly. For the purpose of Physical model building, we introduce a free fermion Lagrangian on $M_4 \times Z_2$ and study Yang-Mills like gauge theory.

1 Introduction

Since Alian Connes introduced non-commutative geometry in particle physics to explain the nature of Higgs field and the symmetry breaking mechanism[1], many efforts have been done alone similar direction[2-6]. It is worthy to mention two interesting approaches, one was given by R.Conquereax, G.Esposito-Farese and G.Vaillant [3], the other by Sitarz and the authors[6,10,11]. Both of them are easy to be understood without entire knowledge of Non-commutative geometry.

Recently,CEV model has been investigated by some authors[7-9], the main idea is generalize the ordinary Yang-Mills potential by a finite matrices. By defining an exterior algebra of forms over the non-commutative direct product algebra of smooth function on spacetime and the Hermitian of the 2×2 matrices, a Lagrangian can be constructed from consideration

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of the generalized two forms. Another was proposed by Sitarz[6], who develop a systematic approach towards the construction of pure gauge theory on arbitrary discrete groups and the authors[10,11] finished the physical model building.

In this paper, based on the gauge theory on discrete group Z_2, we develop a kind of differential calculation on 2×2 matrix algebra and show that all the main formulas in CEV model may be derived from this approach. In other words, the CEV model may be regarded as a matrix representation of the later one. Finally, we discuss how to introduce the coupling between Higgs field and fermion field.

2 Differential Calculation on 2×2 Matrix Algebra

Let Z_2 be a discrete group composed of two elements $Z_2 = \{e, Z | Z^2 = e\}$ and $\mathcal{A}$ be the algebra of complex valued functions on Z_2. Every function f on discrete group Z_2 may be represented as a 2×2 diagonal matrix as follow

$$\mathcal{F} = \begin{pmatrix} f(e) & \\ & f(Z) \end{pmatrix}. \tag{1}$$

We notice that all these matrices construct a algebra $\mathcal{M}$. Using the result of differential calculus on discrete group Z_2, we define $\bar{\partial}_Z$ on the matrices algebra as follow

$$\bar{\partial}_Z = \begin{pmatrix} \partial_Z & \\ & \partial_Z \end{pmatrix} \tag{2}$$

where the action ∂_Z on $f \in \mathcal{A}$ was defined by $\partial_Z f = f - R_Z f$, the right action of the group R_Z acting on f as $(R_Z f)(g) = f(g \odot Z)$, $g \in Z_2$ and $\odot$ is the group multiplication.

It is obviously that one dimension space $\mathcal{E}$ which is spanned by $\bar{\partial}_Z$ forms an algebra

$$\bar{\partial}_Z \cdot \bar{\partial}_Z = 2\bar{\partial}_Z \tag{3}$$

Having the basis of $\mathcal{E}$, we can introduce the basis for the dual space $\mathcal{E}^*$ or $\bar{\Omega}^1$ consisting of forms $\bar{K}$, which satisfy

$$\bar{K}(\bar{\partial}_Z) = I. \tag{4}$$

The definition for the higher order forms is nature and we take $\bar{\Omega}^n$ to be the tensor product of n copies of $\bar{\Omega}^1$, $\bar{\Omega}^n = \bar{\Omega}^{\otimes n}$ and $\bar{\Omega}^0 = \mathcal{M}$. To complete the construction of the differential

algebra $\bar{\Omega}^* = \{\bigoplus_n \bar{\Omega}^n\}$, we need to define the exterior derivative $\bar{d}$, $\bar{d}: \bar{\Omega}^n \to \bar{\Omega}^{n+1}$ whose action on $\mathcal{M}$ is defined by

$$\bar{d}\mathcal{F} = \bar{\partial}_Z \mathcal{F} \bar{K}. \tag{5}$$

If we realized $\bar{d}$ by a matrices operator

$$\bar{d} = i \begin{pmatrix} 0 & d \\ d & 0 \end{pmatrix} \tag{6}$$

where operator d on algebra $\mathcal{A}$ as follow

$$df = \partial_Z f \epsilon, \tag{7}$$

and ϵ is a unit vector, which construct a tensor algebra $\Omega^* = \{\bigoplus_n \Omega^n\}, \Omega^0 = \mathcal{A}, \Omega^n = \Omega^{\otimes n}$. Acting the operator $\bar{d}$ on $\mathcal{F} \in \mathcal{M}$, we have

$$\bar{d}\mathcal{F} = i \begin{pmatrix} & df(e) \\ df(Z) & \end{pmatrix} = i\bar{\partial}_Z \mathcal{F} \begin{pmatrix} & \epsilon \\ \epsilon & \end{pmatrix}. \tag{8}$$

Comparing eq.(5) with (8), we get the matrices representation of $\bar{K}$ with ϵ

$$\bar{K} = \begin{pmatrix} & i\epsilon \\ i\epsilon & \end{pmatrix}.$$

If we define matrix tensor product as follow

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \otimes \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} A \otimes A' + B \otimes C' & A \otimes B' + B \otimes D' \\ C \otimes A' + D \otimes C' & C \otimes B' + D \otimes D' \end{pmatrix}, \tag{9}$$

where $A \cdots D' \in \Omega^*$. We get the matrices representation of forms $\bar{K}^n = \bar{K}^{\otimes n}$ with $\epsilon^{\otimes n}$ as follows:

$$\bar{K}^{2n} = \begin{pmatrix} (i\epsilon)^{2n} & \\ & (i\epsilon)^{2n} \end{pmatrix}, \bar{K}^{2n-1} = \begin{pmatrix} & (i\epsilon)^{2n-1} \\ (i\epsilon)^{2n-1} & \end{pmatrix}. \tag{10}$$

To complete the construction of the differential algebra we need to define the exterior derivative $\bar{d}$ and this is the subject of the following lemma:

There exists exactly one linear exterior derivative operator $\bar{d}$ such that it satisfies

$$\begin{array}{ll} (i) & \bar{d}^2 = 0, \\ (ii) & \bar{d}(A \otimes B) = \bar{d}A \otimes B + (-1)^{DegA} A \otimes \bar{d}B, \quad \forall A, B \in \bar{\Omega}^* \\ (iii) & (\bar{d}\mathcal{F})(V) = V(\mathcal{F}), \quad \forall V \in \mathcal{E}, \mathcal{F} \in \mathcal{M} \end{array} \tag{11}$$

provided that ϵ satisfies the following conditions

$$\begin{array}{l} \epsilon f = f\epsilon, \quad \forall f \in \mathcal{A} \\ d\epsilon = -2\epsilon \otimes \epsilon. \end{array} \tag{12}$$

Using the Lemma, it is easy to show that

$$d\bar{K}^{2n} = 0, \quad d\bar{K}^{2n-1} = -2\bar{K}^{2n}.$$

Then we have

$$\begin{array}{ll} \bar{d}(\mathcal{F}\bar{K}^{2n}) & = \bar{d}\mathcal{F}\bar{K}^{2n} + \mathcal{F}\bar{d}\bar{K}^{2n} \\ & = \begin{pmatrix} & (f(e)-f(Z))(i\epsilon)^{2n+1} \\ (f(Z)-f(e))(i\epsilon)^{2n+1} & \end{pmatrix} \\ \bar{d}(\mathcal{F}\bar{K}^{2n-1}) & = \bar{d}\mathcal{F}\bar{K}^{2n-1} + \mathcal{F}\bar{d}\bar{K}^{2n-1} \\ & = \begin{pmatrix} -(f(e)+f(Z))(i\epsilon)^{2n} & \\ & -(f(Z)+f(e))(i\epsilon)^{2n} \end{pmatrix}, \end{array} \tag{13}$$

which corresponding to the derivative on even and odd matrices of CEV model

$$da_e = i\begin{pmatrix} & a_{22}-a_{11} \\ a_{11}-a_{22} & \end{pmatrix}, \quad da_o = i\begin{pmatrix} a_{21}+a_{12} & \\ & a_{21}+a_{12} \end{pmatrix}. \tag{14}$$

The involution on the differential algebra agrees with the complex conjugation $\mathcal{M}$ and commutes with $\bar{d}$, i.e. $\bar{d}(\bar{\omega})^* = (-1)^{Deg\bar{\omega}}(\bar{d}\bar{\omega})^*$. Again it is sufficient to calculate it if we set the involution $\bar{K}$, the basis of the one forms, we have $\epsilon^* = \epsilon$.

Let us now construct the generalized gauge theory using the above differential forms. We take the gauge transformations to be any proper subset $\mathcal{H} \subset \mathcal{M}$. In particular, we will often take $\mathcal{H}$ to be the group of unitary elements of $\mathcal{M}$

$$\mathcal{H} = \mathcal{U}(\mathcal{M}) = \{a \in \mathcal{M} : aa^\dagger = a^\dagger a = I\}. \tag{15}$$

It is easy to see that the exterior derivative $\bar{d}$ is not covariant with respect to the gauge transformations so that we should introduce the covariant derivative $\bar{d}+\bar{\phi}$, where $\bar{\phi} = \phi\bar{K}, \phi \in \mathcal{M}$ is generalized connection one-form. The requirement that $\bar{d}+\bar{\phi}$ is gauge invariant under the gauge transformations

$$\bar{d}+\bar{\phi} = H(\bar{d}+\bar{\phi})H^{-1}, H \in \mathcal{M}, \tag{16}$$

results in the following transformation rule of $\bar{\phi}$

$$\bar{\phi} \to H\bar{\phi}H^{-1} + H\bar{d}H^{-1} \tag{17}$$

and ϕ transform as

$$\phi \to H\phi \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} H^{-1} \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} + H\bar{\partial}_Z H^{-1}. \tag{18}$$

It is convenient to introduce a new field $\Phi = 1 - \phi$, then (18) is equivalent to

$$\Phi \to H\Phi \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} H^{-1} \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}. \tag{19}$$

It can be shown that the generalized curvature two form

$$\bar{F} = \bar{d}\bar{\phi} + \bar{\phi} \otimes \bar{\phi} \tag{20}$$

is gauge invariant.

In the following, we set $\bar{\phi} = \begin{pmatrix} \phi(e) & \\ & \phi(Z) \end{pmatrix} \cdot \bar{K} = \begin{pmatrix} & i\phi(e)\epsilon \\ i\phi(Z)\epsilon & \end{pmatrix}$. The condition $\bar{\phi} = -\bar{\phi}^\dagger$ enforce the following relation of its coefficients.

$$\phi(Z) = \phi(E)^\dagger. \tag{21}$$

Then we have

$$\bar{F} = \begin{pmatrix} -(\Phi\Phi^\dagger - 1)\epsilon \otimes \epsilon & \\ & -(\Phi^\dagger\Phi - 1)\epsilon \otimes \epsilon \end{pmatrix}. \tag{22}$$

In order to construct the Lagrangian of the gauge theory we need to introduce a metric. Let us define

$$< \epsilon, \epsilon >= \eta, \quad < \epsilon \otimes \epsilon, \epsilon \otimes \epsilon >=< \epsilon, \epsilon >< \epsilon, \epsilon >= \eta^2, \tag{23}$$

then we have the Yang-Mills like Lagrangian of the gauge field

$$\mathcal{L} = -||F||^2 = -Tr < F, F >= -|F_{11}|^2 - |F_{12}|^2 - |F_{21}|^2 - |F_{22}|^2 = -2\eta^2(\Phi\Phi^\dagger - 1)^2 \tag{24}$$

3 Gauge Theory on $M^4 \times Z_2$

In order to get a full Lagrangian of the Higgs field, the kinetic term must be include. To this end, we extend the exterior derivative operator $\bar{d}_{Z_2}$ to the one as follows:

$$\bar{d} = \bar{d}_M + \bar{d}_{Z_2} \tag{25}$$

where $\bar{d}_M = \begin{pmatrix} d_M & \\ & d_M \end{pmatrix}$, d_M is the exterior derivative on spacetime M^4. The nilpotency of $\bar{d}$ requires that

$$\bar{d}_M \bar{d}_{Z_2} = -\bar{d}_{Z_2} \bar{d}_M. \tag{26}$$

Acting $\bar{d}_M \bar{d}_{Z_2}$ and $\bar{d}_{Z_2} \bar{d}_M$ on $f(x) \in \mathcal{M}$ separately, relation (26) is equivalent to

$$\partial_\mu \partial_{Z_2} f(x,g) dx^\mu \otimes \epsilon = -\partial_{Z_2} \partial_\mu f(x,g) \epsilon \otimes dx^\mu, \quad g \in Z_2 \tag{27}$$

thus we have

$$dx^\mu \otimes \epsilon = -\epsilon \otimes dx^\mu. \tag{28}$$

The most general connection one-form on $M^4 \times Z_2$ can be written as

$$\bar{A} = \bar{A}_\mu dx^\mu + \bar{\phi} \bar{K} = \begin{pmatrix} A_\mu(x,e) dx^\mu & i\phi\epsilon \\ i\phi^\dagger \epsilon & A_\mu(x,Z) dx^\mu \end{pmatrix},$$

Using the exterior derivative operator defined by (24), we have

$$\begin{array}{ll} \bar{d}\bar{A} & = \begin{pmatrix} \partial_\mu A_\nu dx^\mu \wedge dx^\nu + (\phi+\phi^\dagger)\epsilon \otimes \epsilon & i(\partial_\mu \phi - (A_\mu - B_\mu)) dx^\mu \otimes \epsilon \\ i(\partial_\mu \phi^\dagger + (A_\mu - B_\mu)) dx^\mu \otimes \epsilon & \partial_\mu A_\nu dx^\mu \wedge dx^\nu + (\phi+\phi^\dagger)\epsilon \otimes \epsilon \end{pmatrix} \\ \bar{A} \otimes \bar{A} & = \begin{pmatrix} A_\mu dx^\mu \wedge A_\nu dx^\nu - \phi\phi^\dagger \epsilon \otimes \epsilon & i(A_\mu \phi - \phi B_\mu) dx^\mu \otimes \epsilon \\ i(B_\mu \phi^\dagger - \phi^\dagger A_\mu) dx^\mu \otimes \epsilon & B_\mu dx^\mu \wedge B_\nu dx^\nu - \phi\phi^\dagger \epsilon \otimes \epsilon \end{pmatrix} \end{array}. \tag{29}$$

where $A = A(x,e), B = A(x,z)$.

The formulas (29) are equavalent to formulas in CEV model, if we write $\mathcal{A} = \begin{pmatrix} A & -i\phi \\ -i\phi^\dagger & B \end{pmatrix}$

$$\bar{d}\mathcal{A} = \begin{pmatrix} dA + (\phi + \phi^\dagger) & i(d\phi - (A - B)) \\ i(d\phi^\dagger + (A - B)) & dB + (\phi + \phi^\dagger) \end{pmatrix},$$

$$\mathcal{A} \otimes \mathcal{A} = \begin{pmatrix} A \wedge A - \phi\phi^\dagger & i(A\phi - \phi B) \\ i(B\phi^\dagger - \phi^\dagger A) & B \wedge B - \phi\phi^\dagger \end{pmatrix}.$$

The generalized curvature two form is

$$F = \bar{d}\bar{A} + \bar{A} \otimes \bar{A} = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix},$$

from the above calculation we have

$$\begin{array}{l} F_{11} = F_{\mu\nu}(x,e) dx^\mu \wedge dx^\nu - (\Phi\Phi^\dagger - 1)\epsilon \otimes \epsilon \\ F_{12} = -iD_\mu \Phi dx^\mu \otimes \epsilon \\ F_{21} = -iD_\mu \Phi^\dagger dx^\mu \otimes \epsilon \\ F_{22} = F_{\mu\nu}(x,Z) dx^\mu \wedge dx^\nu - (\Phi\Phi^\dagger - 1)\epsilon \otimes \epsilon, \end{array}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu],$$

$$D_\mu \Phi = \partial\Phi + A_\mu(x,e)\Phi - \Phi A_\mu(x,Z).$$

$$\Phi = 1 - \phi$$

To get the lagrangian of the pure gauge field, we define the metric as

$$\begin{array}{ccc} < dx^\mu, dx^\nu >= g^{\mu\nu} & , & < \epsilon, \epsilon >= \eta \\ < dx^\mu \otimes \epsilon, dx^\nu \otimes \epsilon >= g^{\mu\nu}\eta & , & < \epsilon \otimes \epsilon, \epsilon \otimes \epsilon >= \eta^2 \end{array} \tag{30}$$

Using the previous metric defination, we can get the largrangian for the gauge field:

$$\mathcal{L} = \frac{1}{N}\{-\frac{1}{4}TrF_{\mu\nu}F^{\mu\nu}(x,e) - \frac{1}{4}TrF_{\mu\nu}F^{\mu\nu}(x,Z) + \eta Tr D_\mu\Phi D^\mu\Phi^\dagger - \eta^2 Tr(\Phi\Phi^\dagger - 1)^2\}. \tag{31}$$

Sometime it's useful to calculate with Dirac gamma matrices. We define a vector space isomorphism map π from the exterior algebra and the Clifford algebra,

$$\pi: \ dx^\mu \to \gamma^\mu, \epsilon \to \gamma^5$$

This is necessary when we want to couple the gauge fields to the spinors.

From previous discussion, the results from this aproach is very similar to those in CEV model, except a form basis ϵ is introduced here.

4 Fermion

A Hilbert space $\bar{H}$ is composed of Dirac spinnor, a vector $\Psi \in \bar{H}$ is defined as follow

$$\Psi = \begin{pmatrix} \psi(x,e) \\ \psi(x,Z) \end{pmatrix} = \begin{pmatrix} \psi_L(x) \\ \psi_R(x) \end{pmatrix}. \tag{32}$$

where

$$\psi(x,e) = \psi_L(x) = \frac{1}{2}(1+\gamma_5)\psi(x), \quad \psi(x,Z) = \psi_R(x) = \frac{1}{2}(1-\gamma_5)\psi(x)$$

are the left and right handed Dirac field respectively.

Then the lagrangian for the free fermion can be written as

$$\mathcal{L}_F = \overline{\Psi} i \bar{\gamma}^i \bar{\partial}_i \Psi, \tag{33}$$

where

$$\bar{\gamma}^i = \begin{cases} \begin{pmatrix} \gamma^\mu & \\ & \gamma^\mu \end{pmatrix}, \mu = 0,1,2,3, \\ \begin{pmatrix} i\gamma^5 & \\ & -i\gamma^5 \end{pmatrix}, \quad i = 5; \end{cases} \tag{34}$$

$$\bar{\partial}_i = \begin{cases} \bar{\partial}_\mu, \quad i = \mu = 0,1,2,3, \\ \mu\bar{\partial}_Z, \quad i = 5, \end{cases}$$

the reason is that

$$\mathcal{L}_F = \overline{\psi}(x)(i\gamma^\mu - \mu)\psi(x).$$

If we require the lagrangian (33) is invariant under the gauge transformation $U \in \mathcal{M}$, we should introduce the covariant derivative D_μ, D_Z, where $\bar{D}_\mu = \bar{\partial}_\mu + igA_\mu$,$\bar{D}_Z = \bar{\partial}_Z + \frac{\lambda}{\mu}\phi\bar{R}_Z$, the convariant derivative on discrete group is definite by the convariant differential forms,

$$\bar{D}_Z f = (\bar{\partial}_Z + \frac{\lambda}{\mu}\phi\bar{K})f = (\bar{\partial}_Z + \frac{\lambda}{\mu}\phi\bar{R}_Z)f\bar{K}, \quad f,\phi \in \mathcal{M},$$

where $\bar{R}_Z = \begin{pmatrix} R_Z & \\ & R_Z \end{pmatrix}$. Then we have

$$\bar{D}_Z = \bar{\partial}_Z + \frac{\lambda}{\mu}\phi\bar{R}_Z$$

and the lagrangian (33) can be written as

$$\begin{aligned} \mathcal{L}_F = \; & \overline{\psi}_L(x)i\gamma^\mu D_{L\mu}\psi_L(x) + \overline{\psi}_R(x)i\gamma^\mu D_{R\mu}\psi_R(x) \\ & -\lambda\overline{\psi}_L(x)\Phi(x)\psi_R(x) - \lambda\overline{\psi}_R(x)\Phi^\dagger(x)\psi_L(x), \end{aligned} \tag{35}$$

where $\Phi = \frac{\mu}{\lambda} - \phi$, and all the coupling in the lagrangian are gauge coupling.

Noticing the fact that

$$\bar{\gamma}^i\bar{D}_i = \bar{\gamma}^i\bar{\partial}_i + \begin{pmatrix} ig_1\gamma^\mu A_\mu(x,e) & i\lambda\gamma^5\phi \\ -i\lambda\gamma^5\phi^\dagger & ig_2\gamma^\mu A_(x,Z)\mu \end{pmatrix}$$

and using the vector space isomorphism of the exterior algebra and Clifford algebra, we have

$$\pi^{-1}: \quad \begin{pmatrix} ig_1\gamma^\mu A_\mu(x,e) & i\lambda\gamma^5\phi \\ -i\lambda\gamma^5\phi^\dagger & ig_2\gamma^\mu A_(x,Z)\mu \end{pmatrix} \to \begin{pmatrix} ig_1 A_\mu(x,e)dx^\mu & \lambda i\phi\epsilon \\ \lambda(i\phi)^\dagger\epsilon & ig_2 A_\mu(x,Z)dx^\mu \end{pmatrix}.$$

Therefore, we introduce the gauge invariant lagrangian for the Boson field parts $\mathcal{L}_{Boson}$ from(31)

$$\mathcal{L}_{Boson} = \frac{1}{N}\{-\frac{g_1^2}{4}TrF_{\mu\nu}F^{\mu\nu}(x,e) - \frac{g_2^2}{4}TrF_{\mu\nu}F^{\mu\nu}(x,Z) + \eta\lambda^2 TrD_\mu\Phi D^\mu\Phi^\dagger - \eta^2\lambda^4 Tr(\Phi\Phi^\dagger - \frac{\mu^2}{\lambda^2})^2\} \tag{36}$$

and the complete lagrangian is then $\mathcal{L} = \mathcal{L}_{Fermion} + \mathcal{L}_{Boson}$.

To conclude, we emphasize that both the two approaches are originate from the work of Connes[1]. In the following paper we will show that gauge theory on discrete group Z_2 equavalent to Connes approach for two discrete points.

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